

MAS231

Assignment 7

Geometry II
Due on 17 March 2008

*Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Post the assignment in the Green Post-Box on the ground floor in the Maths building before **16:00** on Monday.*

1. (a) Compute the first fundamental form F_I of a torus with surface patch σ , U as in question 3 of coursework 6.
(b) Use (a) to find the area of the torus.
2. Consider the cone with $\sigma(u, v) = (u, v, \sqrt{u^2 + v^2})$ as in question 1 of coursework 6.
(a) Find its first fundamental form F_I .
(b) Sketch the curve in the cone given by $\gamma(t) = (e^t \cos(t), e^t \sin(t), e^t)$, $t \in (0, 2\pi)$ and use (a) to show that its length is given by $\sqrt{3}(e^{2\pi} - 1)$.
3. Let $\gamma(t) = (f(t), 0, g(t))$, $t \in (\alpha, \beta)$, be some curve in the x - y plane with $f(t) > 0$. Its ‘surface of revolution’ is defined by the surface patch $\sigma(u, v) = (f(u) \cos(v), f(u) \sin(v), g(u))$, $U = \{(u, v) : (u \in (\alpha, \beta), v \in (0, 2\pi))\}$.
(a) Show that this surface patch is regular if γ is regular.
(b) Find the first fundamental form F_I of σ in terms of f and g .
(c) Sketch the surface for $\gamma(t) = (1/t, 0, t)$ and $t \in (1, \infty)$.
4. *From Last Year's Final Exam!* Let S be the surface defined by

$$S = \{(x, y, z) : z = x^2 + y^2\}.$$

and consider the surface patch for S defined by

$$\tilde{\sigma}(r, \theta) = (r \cos(\theta), r \sin(\theta), r^2), \quad r > 0, 0 < \theta < 2\pi$$

- (a) Find the first fundamental form for the surface patch $\tilde{\sigma}$.
- (b) Use your answer to (a) to determine the surface area of the part of S with $z \leq 2$.